Nirma University
Institute of Technology
Department of Mathematics & Humanities
B. Tech. (ALL) – Semester - I
Calculus (MA101)

Assignment – 2

Part I: Differential Calculus

1. Find n^{th} derivative of $\tan^{-1}(x/a)$.
2. If $(1-x^2)y_2 - xy_1 = 0$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0.$
3. If $y = x^n \log x$, prove that $y_{n+1} = n!/x.$
4. If $I_n = \dfrac{d^n}{dx^n}(x^n.\log x)$, prove that $I_n = nI_{n-1} + (n-1)!$. Hence show that
   $I_n = n!\left[\log x + 1 + \dfrac{1}{2} + \ldots \dfrac{1}{n}\right].$
5. If $y = \left[x + \sqrt{1+x^2}\right]^m$, show that $(x^2+1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0.$
   Hence find $y_n(0)$.
6. $\lim\limits_{x \to 0}\left(\dfrac{1^x + 2^x + 3^x + 4^x}{4}\right)^{1/x}$.
7. $\lim\limits_{x \to 0} \dfrac{a^x \sin bx - b^x \sin ax}{\tan bx - \tan ax}$.
8. If an electric field *E* acts on a liquid or a gaseous polar dielectric, the net dipole moment *p* per unit volume is $P(E) = \dfrac{e^E + e^{-E}}{e^E - e^{-E}} - \dfrac{1}{E}$. Show that $\lim\limits_{E \to 0^+} p(E) = 0$.
9. Find $\lim\limits_{x \to a} \dfrac{\sqrt{2a^3x - x^4} - a\sqrt[3]{a^2x}}{a - \sqrt[4]{ax^3}}$ using L'Hospital Rule. (Marquis de L'Hospital first used the above example to illustrate his rule.)

Part-II Integral Calculus

1. Find $\int_0^a x^a \sqrt[3]{a^6 - x^6}\, dx$, (where '*a*' is constant).
2. Find $\int_{-1}^{1} \sqrt{\dfrac{1-x}{1+x}}\, dx.$
3. Show that $\beta(m,n) = \int_0^1 \dfrac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx.$
4. Trace the Cartesian curves: a) $y^2 = \dfrac{x^2(x+a)}{x-a}$ b) $x^3 + y^3 = 3ax^2$.
5. Trace the following polar curves:
   a) $r = a\sin 2\theta,\ (a>0)$ b) $r = a(1+\sin\theta),\ (a>0)$.