Nirma University
Institute of Technology
Department of Mathematics & Humanities
B. Tech. (ALL) – Semester - I
Calculus (MA101)

Assgnment – 3

Part I: Differential Calculus

1. Show that $(1+x)^x = 1 + x^2 - \frac{1}{2}x^3 + \frac{5}{6}x^4 - \frac{3}{4}x^5 + \frac{33}{40}x^6 + \cdots$
2. Prove that $\log(1 + x + x^2 + x^3 + x^4) = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 - \frac{4}{5}x^5 + \frac{1}{6}x^6 + \cdots$
3. Expand $\cos^{-1}\left(\frac{x-x^{-1}}{x+x^{-1}}\right)$ in ascending powers of x. (x>0).
4. Given $\log_{10} 4 = 0.6021$, calculate approximate value of $\log_{10} 404$.
5. Evaluate the following limits, whichever exists.

a) $\lim_{(x,y)\to(0,0)} \frac{xy \sin xy}{(x^2+y^2)}$

b) $\lim_{(x,y)\to(0,0)} (x^2 + y^2) \sin\frac{1}{xy}$

6. For a function f defined by $f(x,y) = \begin{cases} xy\,\frac{x^2-y^2}{x^2+y^2} & ,for (x,y)\neq(0,0) \\ 0 & ,for (x,y)\neq(0,0) \end{cases}$ Verify whether $f_{yx}(0,0) = f_{xy}(0,0)$.
7. If $z = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right)$, prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{x^2-y^2}{x^2+y^2}$.
8. If $z = 3xy - y^3 + (y^2 - 2x)^{3/2}$, verify that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ and $\frac{\partial^2 z}{\partial x^2}.\frac{\partial^2 z}{\partial y^2} = \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2$.
9. Suppose that your weight ω in pounds is a function $f(c,n)$ of the number c of calories you consume daily and this number n of minutes you exercise daily. Using the units for ω, c *and* n ,interpret in everyday terms the statements
   $\frac{\partial w}{\partial c}(2000,15) = 0.02$ and $\frac{\partial w}{\partial n}(2000,15) = -0.025$
10. A one -meter long bar is heated unevenly, with temperature in ℃ at a distance x meters from one end at a time t given by $H(x,t) = 100e^{-0.1t}\ \sin(\pi x) \quad 0 \le x \le 1$.
    a) Sketch a graph of H against x for t=0 and t=1.
    b) Calculate $H_x(0.2,t)$ *and* $H_x(0.8,t)$. What is the practical interpretation (in terms of temperature) of these two partial derivatives? Explain why each one has the sign it does.
    c) Calculate $H_t(x,t)$. What is its sign? What is its interpretation in terms of temperature?

Part-II Integral Calculus

1. Prove that the area of the loop of the Folium of Descartes: $x^3 + y^3 = 3axy$ is three times the area of one loops of the Lemniscate of Bernoulli: $(x^2 + y^2)^2 = a^2(x^2 - y^2)$.
2. Find the length of the arc of the hyperbolic spiral $r\theta = a$ from the point $r = a \; to \; r = 2a$.
3. Find the length of the arc of the curve $x = e^{\theta}\sin\theta, y = e^{\theta}\cos\theta$ from $\theta = 0 \; to \; \theta = \pi/2$.
4. Show that the length of the loop of the curve $r = a(\theta^2 - 1) \; is \; \frac{8a}{3}$.
5. A steady wind blows a kite due to west. The kite's height above ground from horizontal position $x = 0 \; to \; x = 80 \; ft$ is given by $y = 150 - \frac{1}{40}(x - 50)^2$. Find the distance travelled by the kite.
6. Sketch the region enclosed by the given curve. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width and find the area of the region: $y = \sin x, y = e^x, x = 0, x = \pi/2$.
7. The area cut off from the parabola $\sqrt{x} + \sqrt{y} = 1$ by the line $x + y = 1$ is revolved about this line. Find the volume of the solid generated.
8. A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at a 45° angle at the centre of the cylinder. Find the volume of the wedge.
9. A region between the curve $y = \sqrt{x}, 0 \le x \le 4$, and the x-axis is revolved about the x-axis to generate a solid. Find its volume.
10. Find the area of the surface swept out by revolving the circle $x^2 + y^2 = 1$ about x-axis.