Nirma University
Institute of Technology
Department of Mathematics & Humanities
B. Tech. (ALL) – Semester - I
Calculus (MA101)

Assignment – 4

Part I: Differential Calculus

1. If $u = sin^{-1}\left(\frac{x^3+y^3+z^3}{ax+by+cz}\right)$, Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 2\tan u$.
2. If $z = x^4y^2\, sin^{-1}\left(\frac{x}{y}\right) + \log x - logy$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 6x^4y^2 Sin^{-1}\left(\frac{x}{y}\right)$.
3. If $f(x,y) = x^3y^2 + y\, sinx$ where, x = Sin 2t, y = log t. Find $\frac{df}{dt}$.
4. If $z = x\log xy + y^3$ where $y = sin\,(x^2+1)$. Find $\frac{\partial z}{\partial x}$.
5. If $u = x\emptyset\left(\frac{y}{x}\right) + \varphi\left(\frac{y}{x}\right)$ then show that

   (i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x\emptyset\left(\frac{y}{x}\right)$

   (ii) $x^2u_{xx} + 2xyu_{xy} + u^2u_{yy} = 0$.

Part-II Integral Calculus

1. Evaluate $\iint x + y\ \ dy\, dx$ through the area enclosed by the curves $y = 2x$, $x - y = 2, y = 0, y = 1$.
2. Evaluate $\int_0^{\infty}\int_0^{\infty}(x^2+y^2)\,dx\,dy$ and hence show that $\int_0^{\infty} e^{-x^2} = \sqrt{\pi}/2$.
3. Evaluate $\int_0^1\int_0^{1-x} e^{y/x+y}\,dy\,dx$.
4. Evaluate $\iiint(1+x+y+z)^4\,dz\,dy\,dx$ over the tetrahedron bounded by $x = 0$, $y = 0, z = 0\ \&x + y + z = 1$.
5. Consider the integral $\iint_R x^py^q\,dx\,dy$, where R is the triangle in the xy-plane bounded by $x = 0, y = 0\ \&\ x + y = 1$. Interpret the given integral in terms of Gamma functions .
6. Evaluate $\iint_R x^py^q\,dx\,dy$ where R is the region bounded by $x = 0, y = 0\ \&$ $\frac{x^3}{a} + \frac{y^3}{b} = 1$.
7. Evaluate $\int_0^2\int_0^{4-x^2}\frac{xe^{2y}}{4-y}\,dy\,dx$.
8. Evaluate $\int_0^8\int_{\sqrt[3]{x}}^2\frac{1}{1+y^4}\,dy\,dx$.
9. Evaluate $\int_{-1}^0\int_{-\sqrt{1-x^2}}^0\frac{2}{1+\sqrt{x^2+y^2}}\,dy\,dx$.
10. Evaluate $\int_0^{\pi/4}\int_0^{\log Sec\,v}\int_{-\infty}^{2t} e^x\ dxdtdv$.