Nirma University
Institute of Technology
Department of Mathematics & Humanities
B. Tech. (ALL) – Semester - I
Calculus (MA101)
<u>Assignment – 5</u>

<u>Part I: Differential Calculus</u>

1. If z is a function of x & y and $x = e^u + e^{-v}, y = e^{-u} + e^v$ prove that
$$x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} - \frac{\partial z}{\partial v}.$$

2. If V is a function of u, v where $u = x - y$ and $v = xy$ prove that
$$x\frac{\partial^2 V}{\partial x^2} + y\frac{\partial^2 V}{\partial y^2} = (x + y)(\frac{\partial^2 V}{\partial u^2} + xy\frac{\partial^2 V}{\partial v^2}).$$

3. Find Taylor's expansion of $f(x, y) = cos^{-1}xy$ in powers of $(x + 0.5)$ and $(y - 2)$ up to second degree terms. Hence compute $f(-0.4, 2.2)$ approximately.

4. An aquarium with rectangular sides and bottom (and no top) is to hold 32 litres water. Find its dimensions so that it will use the least amount of material.

5. Divide 24 into three parts such that the continued product of the first, square of the second and the cube of the third may be maximum.

<u>Part-II Integral Calculus</u>

1. Evaluate $\iint (y - x) \, dx \, dy$ over the region enclosed by the straight lines $y = x + 1$,
$$y = x - 3, y = -\frac{1}{3}x + \frac{7}{3}, y = -\frac{1}{3}x + 5.$$

2. Calculate the area which is inside the cardioid $r = 2(1 + cos\theta)$ and outside the circle $r = 2$.

3. Find the volume of the tetrahedron bounded by the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and the coordinate planes.

4. Evaluate the volume of the sphere of radius 'a' using triple integrals.