<u>Tutorial – 3</u>

Part I: Differential Calculus

1. Find n^{th} derivative of $\tan^{-1}(x/a)$.

2. If $(1-x^2)y_2 - xy_1 = 0$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$.

3. If $y = x^n \log x$, prove that $y_{n+1} = n!/x$.

4. If $I_n = \dfrac{d^n}{dx^n}(x^n . \log x)$, prove that $I_n = nI_{n-1} + (n-1)!$. Hence show that

$$I_n = n!\left[\log x + 1 + \frac{1}{2} + \dots \frac{1}{n}\right].$$

5. If $y = \left[x + \sqrt{1+x^2}\right]^m$, show that $(x^2+1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$. Hence find $y_n(0)$.

Part-II Integral Calculus

1. Find $\displaystyle\int_0^a x^a \sqrt[3]{a^6 - x^6}\, dx$, (where 'a' is constant).

2. Find $\displaystyle\int_{-1}^{1} \sqrt{\frac{1-x}{1+x}}\, dx$.

3. Show that $\beta(m,n) = \displaystyle\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$.

4. Evaluate $\displaystyle\int_0^1 \frac{x^{2n}}{\sqrt{1-x^2}}\, dx$.

5. Evaluate $erf(x) + erf_c(-x)$