

Nirma University
Institute of Technology
Department of Mathematics & Humanities
B. Tech. (ALL) – Semester - I
Calculus (MA101)
Tutorial – 3

Part I: Differential Calculus

1. Find n^{th} derivative of $\tan^{-1}(x/a)$.
2. If $(1-x^2)y_2 - xy_1 = 0$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$.
3. If $y = x^n \log x$, prove that $y_{n+1} = n!/x$.
4. If $I_n = \frac{d^n}{dx^n}(x^n \cdot \log x)$, prove that $I_n = nI_{n-1} + (n-1)!$. Hence show that
$$I_n = n! \left[\log x + 1 + \frac{1}{2} + \dots + \frac{1}{n} \right].$$
5. If $y = [x + \sqrt{1+x^2}]^m$, show that $(x^2+1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$.
Hence find $y_n(0)$.

Part-II Integral Calculus

1. Find $\int_0^a x^a \sqrt[3]{a^6 - x^6} dx$, (where $a \neq 0$ is constant).
2. Find $\int_{-1}^1 \sqrt{\frac{1-x}{1+x}} dx$.
3. Show that $\beta(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$.
4. Evaluate $\int_0^1 \frac{x^{2n}}{\sqrt{1-x^2}} dx$.
5. Evaluate $\operatorname{erf}(x) + \operatorname{erf}_c(-x)$