Nirma University
Institute of Technology
Department of Mathematics & Humanities
B. Tech. (ALL) – Semester - I
Calculus (MA101)

Given week: **Tutorial – 6** **Submission week:**

Part I: Differential Calculus

1. If $z = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$.

2. If $z = 3xy - y^2 + (y^2 - 2x)^{3/2}$, verify that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ and $\frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} = \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2$.

3. Suppose that your weight ω in pounds is a function $f(c, n)$ of the number c of calories you consume daily and this number n of minutes you exercise daily. Using the units for ω, c and n, interpret in everyday terms the statements
$\frac{\partial w}{\partial c}(2000,15) = 0.02$ and $\frac{\partial w}{\partial n}(2000,15) = -0.025$

4. A one -meter long bar is heated unevenly, with temperature in $^{\circ}C$ at a distance x meters from one end at a time t given by $H(x,t) = 100e^{-0.1t} \sin(\pi x) \quad 0 \le x \le 1$.

   a) Calculate $H_x(0.2,t)$ and $H_x(0.8,t)$. What is the practical interpretation (in terms of temperature) of these two partial derivatives? Explain why each one has the sign it does.

   b) Calculate $H_t(x,t)$. What is its sign? What is its interpretation in terms of temperature?

Part-II Integral Calculus

1. A steady wind blows a kite due to west. The kiteôs height above ground from horizontal position $x = 0$ to $x = 80$ ft is given by $y = 150 - \frac{1}{40}(x - 50)^2$. Find the distance travelled by the kite.

2. Sketch the region enclosed by the given curve. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width and find the area of the region: $y = \sin x, y = e^x, x = 0, x = \pi/2$.

3. A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at a 45° angle at the centre of the cylinder. Find the volume of the wedge.

4. A region between the curve $y = \sqrt{x}, 0 \le x \le 4$, and the x-axis is revolved about the x-axis to generate a solid. Find its volume.

5. Find the area of the surface swept out by revolving the circle $x^2 + y^2 = 1$ about x-axis.