Nirma University
Institute of Technology
Department of Mathematics & Humanities
B. Tech. (ALL) – Semester - I
Calculus (MA101)

Given week: **Tutorial – 7** **Submission week:**

Part I: Differential Calculus

1. If $u = sin^{-1}\left(\frac{x^3+y^3+z^3}{ax+by+cz}\right)$, Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=2\tan u$.
2. If $z = x^4y^2\ sin^{-1}\left(\frac{x}{y}\right)+\log x-logy$, show that $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=6x^4y^2Sin^{-1}\left(\frac{x}{y}\right)$.
3. If $u = x\phi\left(\frac{y}{x}\right)+\varphi\left(\frac{y}{x}\right)$ then show that
   (i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=x\phi\left(\frac{y}{x}\right)$
   (ii) $x^2u_{xx}+2xyu_{xy}+u^2u_{yy}=0$.

Part-II Integral Calculus

1. Evaluate $\iint x+y\ dy\,dx$ through the area enclosed by the curves $y = 2x$, $x-y=2, y=0, y=1$.
2. Evaluate $\int_0^\infty \int_0^\infty (x^2+y^2)dx\,dy$ and hence show that $\int_0^\infty e^{-x^2}=\frac{\sqrt{\pi}}{2}$.
3. Evaluate $\int_0^1 \int_0^{1-x} e^{y/x+y}\,dy\,dx$.